Week 11 : 7th Dec 2015 Jed Gibbs for Charlton Rodda

FOUNDATIONS OF COMPUTER SCIENCE

Lectures 35 + 36 - Pavel Sobocinski

Soundness - do the formal symbols we use match the underlining meaning - the 'semantics' of the problem.

Completeness - is it ~~provable~~ possible to prove everything in that is semantically intended.

Gentzen - Natural deduction proof system.

Syntax vs Semantics

Martian example - if he observed a translator converting strings of words from English to French

$$\begin{tikzcd}
\text{ENGLISH} \arrow[rr, "\substack{\text{like for like translation}\\ \text{(syntax)}}"] \arrow[dr] & & \text{FRENCH} \\
& \substack{\text{MEANING}\\ \text{(semantics)}} \arrow[ur] &
\end{tikzcd}$$

The 'syntax' version should match with the 'semantics' version

PROOFS IN NATURAL DEDUCTION

see slide 7 for a diagram

- 'leaves' are assumptions which can be used as many times as you need.
- 'root' is proved formula
- internal ~~nodes~~ nodes are determined by application of 'proofrules'

Proof Rules:

introduction rules - allow intro of a logical connective
elimination rules - " elimination " " " "

<u>Logical Connection rules</u>

There is 1
$\wedge$ - introduction rule

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi}$$

There are 2
$\wedge$ - elimination rules

$$\frac{\varphi \wedge \psi}{\varphi} \qquad \frac{\varphi \wedge \psi}{\psi}$$

slide

Rules for 'Implies' - see slide

Example - works through slide 10 to prove χ with assumptions $\varphi \wedge \psi$, $\varphi \rightarrow (\psi \rightarrow \chi)$. (The workings on the board are not clear enough to transcribe reliably)

Slide 12 - Discharging Assumptions

$\wedge$ - introduction

$$\frac{\varphi \quad \psi}{\varphi \wedge \psi}$$

see slide to explain the formula approaches ~~tautology~~

sometimes $\neg \varphi$ is understood as shorthand for $\varphi \rightarrow \bot$

Rules for 'OR'

Slide 14.

Example slide 15   Ex (Truss 7.1 (3))

proof of χ with assumptions φ and $(\varphi \wedge \neg\varphi) \vee (\varphi \wedge \chi)$

next example on slide 16 is 'homework'

Rules for 'FALSE' - slide 17

$$\frac{\bot}{\varphi}$$

The second rule is 'non-constructive'
intuitionistic logic rules it out

$$\begin{array}{c}[\neg\varphi] \\ \vdots \\ \dfrac{\bot}{\varphi}\end{array}$$

Slide 18 has all
the rules and will be supplied for the exam on paper.

Slide 19 - FORMAL PROOFS

recall - syntax trees
- 'internal nodes' labelled with operations $(\neg, \wedge, \vee, \rightarrow)$
- 'leaves' labelled* with propositional variables (p, r, s) or $\bot$ constant

slide 20   '$\models$' semantically implies
'$\vdash$' proves

Soundness Theorem $\Gamma \vdash \varphi$ implies $\Gamma \models \varphi$
There is a handout for this which will be put on the homepage.

Pavel then asks the group to invent a non-sound natural deduction style proof rule.

Prove that it is not sound

Ideas sought for an algorithm that can print out ALL the tautologies of propositional logic.

The proofs are recursive ~~sets~~ objects.

Tautology is a proof with no assumptions